Error Analysis of Statistical Linearization with Gaussian Closure for Large Degree-of-Freedom Systems

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STRUCTURAL RELIABILITY THEORY
PAPER NO. 140

Submitted to "Probabilistic Engineering Mechanics"

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ABSTRACT

This paper contains an analysis of the error induced by applying the method of equivalent statistical linearization (ESL) to randomly-excited multi-degree-of-freedom (MDOF) geometrically nonlinear shear-frame structures as the number of degrees of freedom increases. The quantity that is analyzed is the variance of the top-story displacement. The MDOF systems under consideration obtain their nonlinearity through cubic polynomial interstory restoring forces and the external excitation is modeled as the stationary output of a Kanai-Tajimi filter (which itself is excited by Gaussian white noise). Parameters of the filter and the MDOF structures, as well as the intensity of the Gaussian white noise, are calibrated such that quantitative comparisons of the error between the exact solutions, estimated from Monte Carlo simulations, and the ESL solutions are possible among systems of different dimensions.

1. INTRODUCTION

Since its introduction[1,2,3] the method of equivalent statistical linearization has become a popular means of analyzing nonlinear systems subject to random excitation. The essential idea of ESL is that the nonlinear system under consideration is replaced by an equivalent linear system where the coefficients of the linear system are calibrated by minimizing (in some way) the difference between the actual nonlinear system and the equivalent linear system. Unlike other methods for analyzing nonlinear random-vibrations problems (e.g., the Fokker-Planck equation, perturbation methods), ESL is relatively easy to implement, is computationally efficient, and is subject to few constraints. In the past few decades, much work in the field of random vibrations has centered around refining the method and extending it to various specific cases[7,8,11,15] for example MDOF systems[4] and systems subject to nonstationary excitation[5].

While generally considered applicable to a wide array of random-vibrations problems, the method of ESL does have a few inherent drawbacks. For instance, the way in which the difference between the actual and approximate systems is minimized is arbitrary. The usual convention is that the mean-square of the difference is minimized although other minimization procedures are equally viable. Another inherent problem with the ESL method is that in order to determine the coefficients of the equivalent linear system, the response statistics of the equivalent linear system are used which in turn depend on the coefficients of the equivalent linear system. Thus, a set of nonlinear equations for the calibration of the linear coefficients arises. Moreover, for nonlinear systems subject to Gaussian white noise, the method of ESL predicts a Gaussian response, whereas the actual response may be decidedly non-Gaussian. Consequently, the response spectral density corresponding to the ESL solution will contain only frequencies inherent in the excitation, while the actual response spectral density for the nonlinear system may exhibit frequencies outside the spectrum of the excitation[14] . In addition, as shown by Fan and Ahmadi[10] and Roberts[9], the uniqueness of ESL solutions is not always guaranteed.

However, despite the aforementioned drawbacks, the method of ESL remains the most widely-used method in the analysis of randomly-excited nonlinear MDOF systems. In this regard, it has been known for some time that the absolute error of the stochastic response grows with the number of degrees of freedom of the system, yet it appears as though no systematic analysis of this problem has been reported in the literature. The objective of this paper, then, is to attempt to quantify the growth in the error of the mean-square top-story displacement for an n-degree-of-freedom system as n becomes large. The basis for the analysis is an n-DOF shear-frame structure with nonlinear interstory restoring forces subject to stationary random excitation. The stationary excitation is modelled as the output of a Kanai-Tajimi filter driven by stationary Gaussian white noise. In order to make meaningful comparisons among structures of different dimensions, the intensity of the Gaussian white noise is chosen such that when the structure is excited in its first mode of vibration, the top-story displacement variance is $0.2n$ meters for an n-DOF structure. In this manner, the interstory displacements for structures of varying degrees of freedom are roughly comparable. Moment equations for the equivalent linear system are derived via Itô's equation and integrated to stationarity using a variable-time-step, error-monitoring Runge-Kutta scheme to insure stability and accuracy. Since the Fokker-Planck equation has not been solved for the actual MDOF nonlinear systems under consideration, the "exact" solutions are estimated from Monte Carlo simulations, whereupon error estimates between the exact and approximate results are calculated.

2. PROBLEM FORMULATION

Consider the n-DOF nonlinear shear frame structure depicted in Figure 1. All stories of the structure are assumed to have the same mass, m, while all interstories are assumed to have the same stiffness and damping parameters, k and c, respectively. The relative displacement between stories i and $i-1$ is denoted by $X_i(t)$. Thus, the top-story displacement relative to the ground surface, X_0, is given by $Y(t) = \sum_{i=1}^{n} X_i(t)$. The structure obtains its nonlinearity through interstory shear restoring forces, which are assumed to be given by the cubic elastic expression $k(X_i(t) + \epsilon X_i^3(t))$ where ϵ is a parameter that controls the magnitude of the nonlinearity (assumed to be the same for each interstory). The ground immediately below the structure and above the bedrock is modelled as a linear shear oscillator with mass m_s, stiffness k_s, and damping constant c_s, through which stationary Gaussian white noise, $\ddot{u}_b$, is filtered .

With the above considerations, the equations of motion governing the combined structure-filter system are given by

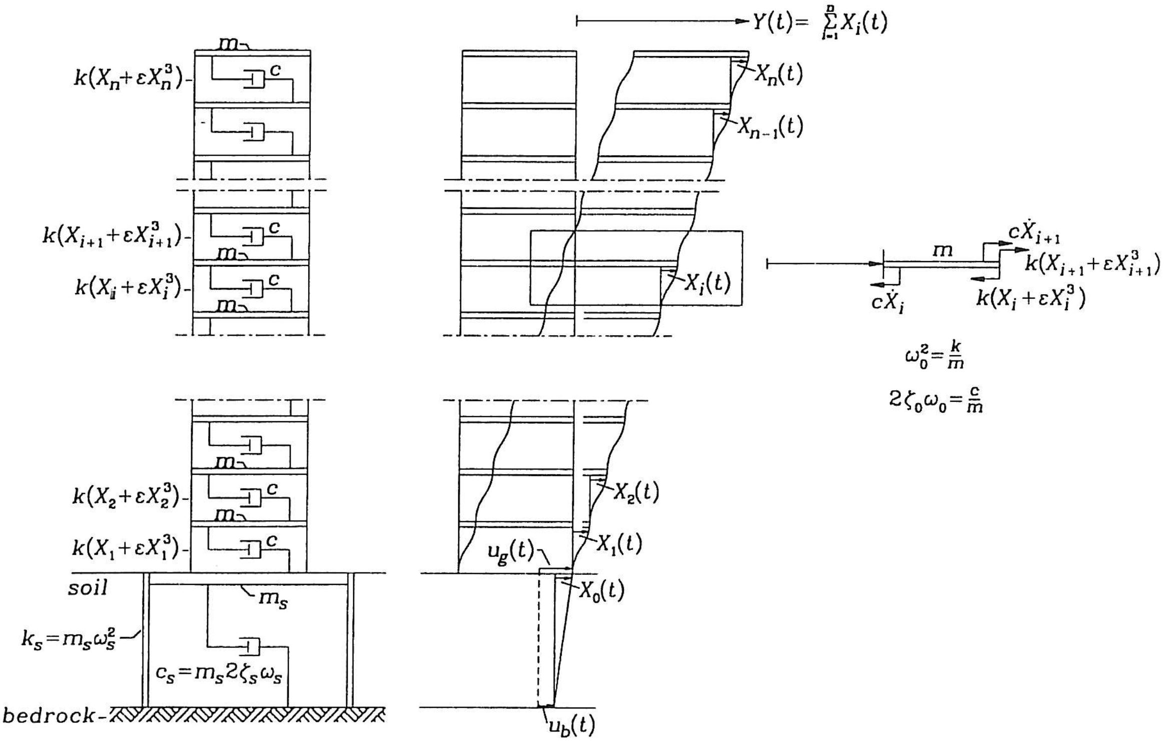

Figure 1: Multi-degree-of-freedom shear-frame structure in series with Kanai-Tajimi filter.

$$
\begin{aligned}
m_s(\ddot{u}_b + \ddot{X}_0) &= -k_s X_0 - c_s \dot{X}_0 + k(X_1 + \epsilon X_1^3) + c\dot{X}_1 \\
m(\ddot{u}_b + \ddot{X}_0 + \ddot{X}_1) &= -k(X_1 + \epsilon X_1^3) - c\dot{X}_1 + k(X_2 + \epsilon X_2^3) + c\dot{X}_2 \\
m(\ddot{u}_b + \ddot{X}_0 + \ddot{X}_1 + \ddot{X}_2) &= -k(X_2 + \epsilon X_2^3) - c\dot{X}_2 + k(X_3 + \epsilon X_3^3) + c\dot{X}_3 \\
&\vdots \\
m(\ddot{u}_b + \ddot{X}_0 + \ddot{X}_1 + \cdots + \ddot{X}_{n-1}) &= -k(X_{n-1} + \epsilon X_{n-1}^3) - c\dot{X}_{n-1} + k(X_n + \epsilon X_n^3) + c\dot{X}_n \\
m(\ddot{u}_b + \ddot{X}_0 + \ddot{X}_1 + \ldots + \ddot{X}_{n-1} + \ddot{X}_n) &= -k(X_n + \epsilon X_n^3) - c\dot{X}_n
\end{aligned}
\tag{1}
$$

In what follows, k_s, c_s, k, and c are expressed as $m_s\omega_s^2$, $2m_s\zeta_s\omega_s$, $m\omega_0^2$, and $2m\zeta_0\omega_0$, respectively. Here ω_s and ζ_s represent the circular eigenfrequency and damping ratio, respectively, of the linear single-degree-of-freedom (SDOF) soil filter. The values ω_0 and ζ_0 as defined above are simply parameters of the MDOF system. (For a linear SDOF system, ω_0 corresponds to the first circular eigenfrequency of the structure and similarly, ζ_0 corresponds to the structure's damping ratio, but this is not the case for MDOF systems.) In addition, the mass of the soil is assumed to be considerably greater than the mass of the stories.

By dividing each of equations (1) by their respective mass terms and subtracting the $(i-1)$-th equation from the i-th, the system of equations can be written in matrix form as

$$\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}(\mathbf{X}(t) + \epsilon\mathbf{X}^3(t)) = -\mathbf{U}\ddot{u}_b(t) \tag{2}$$

where

$$\mathbf{X}(t) = \begin{bmatrix} X_0(t) \\ X_1(t) \\ \vdots \\ X_n(t) \end{bmatrix}, \quad \mathbf{X}^3(t) = \begin{bmatrix} X_0^3(t) \\ X_1^3(t) \\ \vdots \\ X_n^3(t) \end{bmatrix}, \quad \mathbf{U} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{3}$$

$$\mathbf{C} = 2\zeta_0\omega_0 \begin{bmatrix} \frac{\zeta_s\omega_s}{\zeta_0\omega_0} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ -\frac{\zeta_s\omega_s}{\zeta_0\omega_0} & 1 & -1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & -1 & 2 \end{bmatrix} \tag{4}$$

$$\mathbf{K} = \omega_0^2 \begin{bmatrix} \frac{\omega_s^2}{\omega_0^2} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ -\frac{\omega_s^2}{\omega_0^2} & 1 & -1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & -1 & 2 \end{bmatrix}, \quad \boldsymbol{\epsilon} = \epsilon \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

It should be noted that the last two terms of the first of Eqs. (1) drop out due to the assumption that m is small compared to m_s. This assumption entails that there is no feedback from the structure to the soil. Thus, the soil oscillator acts as a Kanai-Tajimi filter[12]. The input to the Kanai-Tajimi filter, the bedrock excitation $\ddot{u}_b$, is taken as stationary Gaussian white noise with autocorrelation given by

$$\mathrm{E}[\ddot{u}_b(t)\ddot{u}_b(t+\tau)] = d_0^2\delta(\tau) \tag{5}$$

where d_0 is the intensity of the white noise. While a stationary excitation process is not realistic, the error analysis as it is conducted herein remains valid for any type of excitation, therefore, the simplest excitation model is adopted.

3. MOMENT EQUATIONS FOR EQUIVALENT LINEAR SYSTEMS

The theory of vector Markov diffusion processes can be applied to equations (2) to yield the following stochastic differential equations in state-space form

$$d\mathbf{Z}(t) = \mathbf{a}(\mathbf{Z}(t))dt + \mathbf{b}dW(t) \quad , \quad \mathbf{Z}(0) = \mathbf{0} \tag{6}$$

where

$$\mathbf{Z}(t) = \begin{bmatrix} \mathbf{X}(t) \\ \dot{\mathbf{X}}(t) \end{bmatrix} \; , \; \mathbf{a}(\mathbf{Z}(t)) = \begin{bmatrix} \dot{\mathbf{X}}(t) \\ -\mathbf{C}\dot{\mathbf{X}} - \mathbf{K}(\mathbf{X} + \epsilon\mathbf{X}^3(t)) \end{bmatrix} \; , \; \mathbf{b} = \begin{bmatrix} \mathbf{0} \\ \mathbf{U}d_0 \end{bmatrix} \tag{7}$$

and W(t)is a unit Wiener process.

Equation (6) is nonlinear as the drift vector is a function of the elements of the state vector $\mathbf{Z}(t)$. A statistical linearization approximation to (6) can be written in the following form:

$$d\mathbf{Z}(t) = \mathbf{A}(t)\mathbf{Z}(t) + \mathbf{b}dW(t) \quad , \quad \mathbf{Z}(0) = \mathbf{0} \tag{8}$$

where

$$\mathbf{A}(t) = E\left[\frac{\partial}{\partial \mathbf{Z}}\mathbf{a}(\mathbf{Z}(t))\right] = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{K}\left(\mathbf{I} + 3\epsilon\boldsymbol{\sigma}^2_{\mathbf{X}}(t)\right) & -\mathbf{C} \end{bmatrix} \tag{9}$$

Here $\boldsymbol{\sigma}^2_{\mathbf{X}}(t)$ is a diagonal matrix with entries $\sigma^2_{X_i}(t)$ where $\sigma^2_{X_i}(t)$ is the the i-th interstory displacement variance of the approximating linear system. The expression for $\mathbf{A}(t)$ is found by a mean least-squares estimate between the actual and approximating systems where the expectations are evaluated using Gaussian closure[5,6].

Itô's differential formula can be used to derive differential equations for the joint statistical moments of response of the equivalent linear system. Since the drift vector $\mathbf{a}(\mathbf{Z}(t))$ (and consequently the product $\mathbf{A}(t)\mathbf{Z}(t)$) fulfills the asymmetry condition $\mathbf{a}(\mathbf{Z}(t)) = -\mathbf{a}(-\mathbf{Z}(t))$, the mean of the state vector is zero, i.e., $\mathrm{E}[\mathbf{Z}(t)] = \mathbf{0}$. Thus, as the moment equations for a linear system are closed, only equations for the covariances $\kappa_{ij}(t) = \mathrm{E}[Z_i(t)Z_j(t)]$ need to be formulated. These equations can be written in matrix form as

$$\frac{d}{dt}\boldsymbol{\kappa}(t) = \mathbf{A}(t)\boldsymbol{\kappa}(t) + \boldsymbol{\kappa}(t)\mathbf{A}^T(t) + \mathbf{d}\mathbf{d}^T \quad , \quad \boldsymbol{\kappa}(0) = \mathbf{0} \tag{10}$$

Equation (10) is nonlinear since $\mathbf{A}(t)$ depends on $\boldsymbol{\kappa}(t)$ (i.e., $\sigma^2_{X_i}(t) = \kappa_{ii}(t)$). The stationary covariance matrix can be found either by setting the left-hand sides of (10) to zero and solving the resulting nonlinear algebraic equations or by integrating equations (10) to stationarity. Here the latter approach was taken, employing a variable time-step, error-monitoring 4th-order Runge-Kutta scheme[13] to ensure accuracy and stability.

4. NUMERICAL EXAMPLE

In order to make meaningful comparisons of error estimates among structures of different dimensions for given values of ω_s, ζ_s, and ϵ, two steps were taken to ensure consistent response behavior independent of the number of degrees of freedom . As the first excitation mode is the dominant response mode for shear-frame structures, the first step was to select the n-DOF system parameters $\omega_0 = \omega_0(n)$ and $\zeta_0 = \zeta_0(n)$ so that ω_1 and ζ_1 were the same for all structures where ω_1 and ζ_1 denote the corresponding linear structure's first circular eigenfrequency and first modal damping ratio, respectively. The parameters $\omega_0(n)$ and $\zeta_0(n)$ are related to the ω_1 and ζ_1 in the following manner

$$\omega_0(n) = \frac{\omega_1}{(\lambda_1(n))^{\frac{1}{2}}} \quad , \quad \zeta_0(n) = \frac{\zeta_1}{(\lambda_1(n))^{\frac{1}{2}}} \tag{11}$$

where $\lambda_1(n)$ is the first eigenvalue of the $n \times n$ non-dimensional stiffness matrix $\mathbf{k}$ given by

$$\mathbf{k} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & -1 & 2 \end{bmatrix} \tag{12}$$

Values of $2\pi\text{s}^{-1}$ and 0.05 were taken for ω_1 and ζ_1, respectively.

The second step involved the calibration of ground motion intensities at first-mode resonant excitation, for which interstory displacements are fully correlated (see Table 1) and identically distributed. In this case, the standard deviation of the top-story displacement for an n-DOF equivalent linear structure is given by $\sigma_{Y,eq}(n) = n\sigma_{X_1,eq}$. Thus, by selecting the ground motion intensities so that $\sigma_{Y,eq}(n) = n\Delta$ where Δ is an arbitrary constant, the standard deviations of interstory displacements are approximately the same at stationarity (i.e., $\sigma_{X_i} \approx \Delta$) regardless of the number of degrees of freedom. Consequently, the equivalent linear approximation to the actual nonlinear system given by the term $3\epsilon\sigma^2_{X_i}(t)$ will also be approximately the same for all structures. In the present analysis, Δ was taken as 0.2m while ϵ was taken, respectively, as 2.0 m^{-2} and 8.0m^{-2} corresponding to values of $3\epsilon\sigma^2_{X_i}(t)$ equal to 0.24 (moderate nonlinearity) and 0.96 (severe nonlinearity).

Table 1. Correlation coefficient matrices $\rho_{X_i X_j}$ as functions of ω_s and ζ_s. for $n = 3$, $\epsilon = 0$ (linear case)

$\begin{bmatrix} 1.000 & 1.000 & 1.000 \\ & 1.000 & 1.000 \\ . & & 1.000 \end{bmatrix}$	$\begin{bmatrix} 1.000 & 1.000 & 1.000 \\ & 1.000 & 1.000 \\ & & 1.000 \end{bmatrix}$	$\begin{bmatrix} 1.000 & 1.000 & 0.998 \\ & 1.000 & 1.000 \\ & & 1.000 \end{bmatrix}$
$\zeta_s = 0.30,\ \omega_s = \omega_1$	$\zeta_s = 0.10,\ \omega_s = \omega_1$	$\zeta_s = 0.01,\ \omega_s = \omega_1$

Table 2 lists the first circular eigenfrequencies for n-DOF structures as well as the calibrated ground motion intensities for fixed values of nonlinear parameter, ϵ, and soil damping, ζ_s. Using the calibrated ground motion intensities, error estimates were found for various values of ω_s/ω_1 as functions of ϵ and ζ_s.

Table 2: Eigenvalues λ_1 and acceleration intensities d_0 as a function of n, ϵ and ζ_s for $\omega_s = \omega_1 = 2\pi\ s^{-1}$

n	$\lambda_1(n)$	$d_0(n)[m/s^2]$, $\epsilon = 2.0\ m^{-2}$			$d_0(n)[m/s^2]$, $\epsilon = 8.0\ m^{-2}$		
	$[10^{-2}]$	$\zeta_s = 0.30$	$\zeta_s = 0.01$	$\zeta_s = 0.10$	$\zeta_s = 0.30$	$\zeta_s = 0.10$	$\zeta_s = 0.01$
1	100.00	0.942	0.481	0.158	1.785	1.400	0.581
2	38.20	1.298	0.702	0.280	2.617	2.259	1.062
3	19.81	1.610	0.910	0.415	3.370	3.080	1.580
4	12.06	1.895	1.100	0.550	4.050	3.820	2.100
5	8.10	2.150	1.280	0.680	4.660	4.560	2.630
6	5.81	2.380	1.436	0.804	5.220	5.140	3.110
7	4.37	2.590	1.580	0.922	5.725	5.720	3.590
8	3.41	2.780	1.712	1.030	6.200	6.250	4.050
9	2.73	2.970	1.845	1.142	6.650	6.770	4.510
10	1.03	3.160	1.980	1.260	7.120	7.300	4.980

Figures 2(a)-2(f) show the absolute relative error between exact and approximate results as a function of ω_s/ω_1 for $n = 4, 6, 8,$and 10. For each plot, both the nonlinear parameter and the soil damping ratio are fixed.

For moderate nonlinearity and wide-band excitation, Figure 2(a), the largest relative errors occur at $\omega_s/\omega_1 = 2$, presumably the resonance condition of the equivalent linear structure (as the equivalent linear restoring force is greater than the restoring force of the corresponding linear system ($\epsilon = 0$), the fundamental frequency of the equivalent linear structure is higher than the fundamental frequency of the corresponding linear structure, namely $\omega_1 = 2\pi s^{-1}$). Thus, when the energy of the input excitation is concentrated

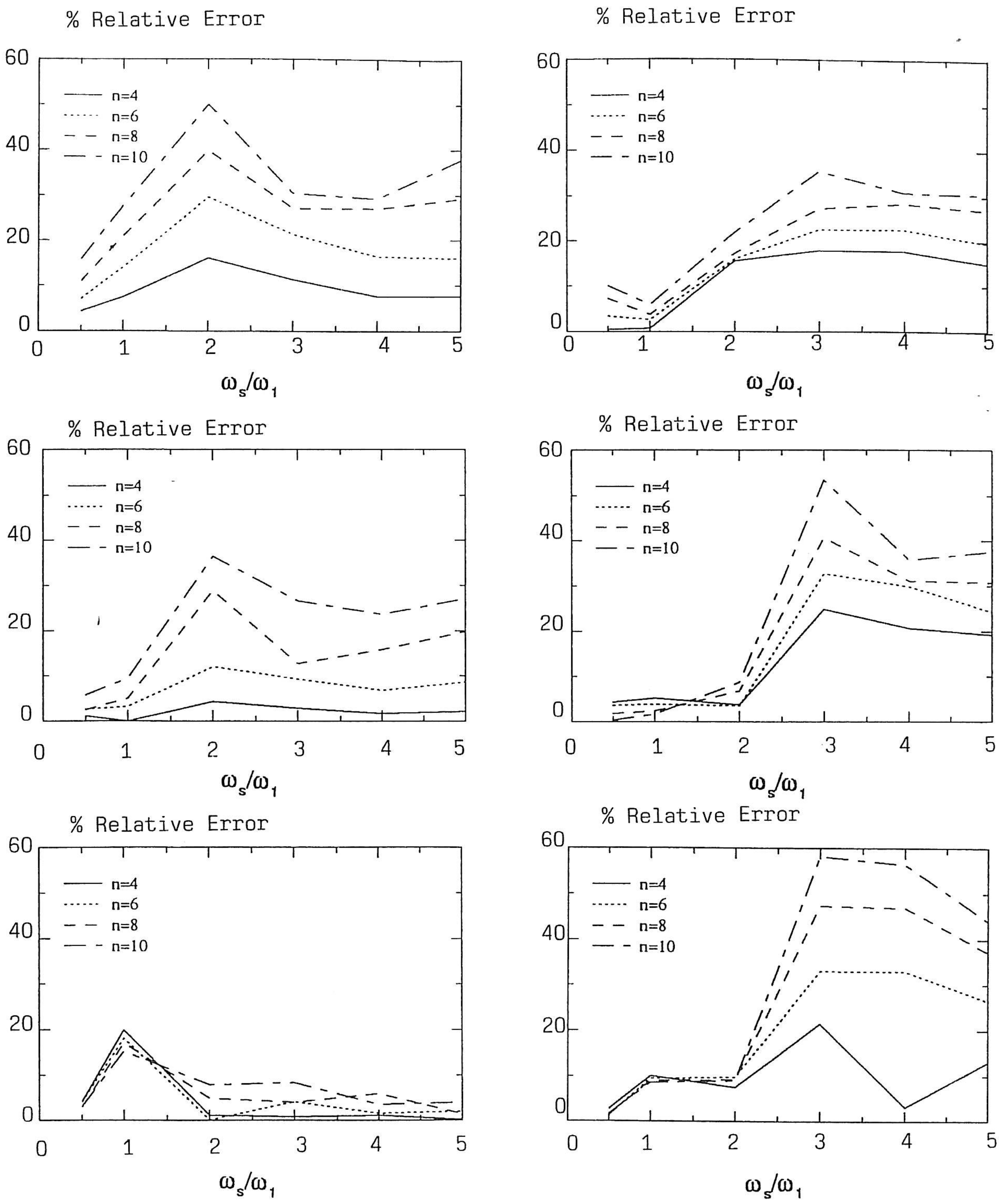

Figure 2: The percent relative error between exact and ESL solutions for the top-story displacement variance as a function of ω_s/ω_1. (a) $\epsilon = 2.0\text{m}^{-2}$, $\zeta_s = 0.30$; (b) $\epsilon = 8.0\text{m}^{-2}$, $\zeta_s = 0.30$; (c) $\epsilon = 2.0\text{m}^{-2}$, $\zeta_s = 0.10$; (d) $\epsilon = 8.0\text{m}^{-2}$, $\zeta_s = 0.10$; (e) $\epsilon = 2.0\text{m}^{-2}$, $\zeta_s = 0.01$; (f) $\epsilon = 8.0\text{m}^{-2}$, $\zeta_s = 0.01$.

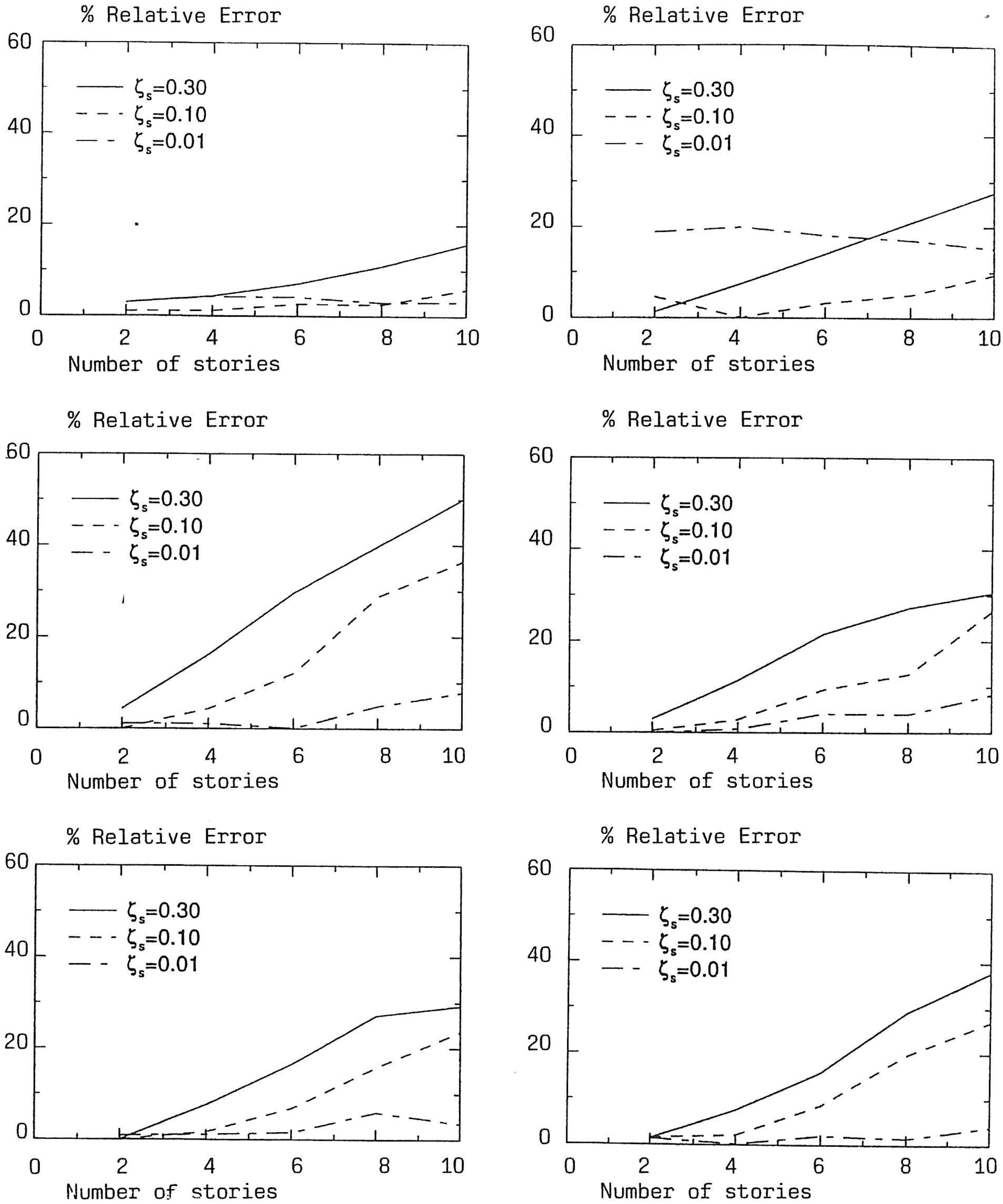

Figure 3: The percent relative error between exact and ESL solutions for the top-story displacement variance as a function of the number of shear-frame stories: moderate nonlinearity (a) $\epsilon = 2.0\text{m}^{-2}$, $\omega_s/\omega_1 = 0.5$; (b) $\epsilon = 2.0\text{m}^{-2}$, $\omega_s/\omega_1 = 1.0$; (c) $\epsilon = 2.0\text{m}^{-2}$, $\omega_s/\omega_1 = 2.0$; (d) $\epsilon = 2.0\text{m}^{-2}$, $\omega_s/\omega_1 = 3.0$; (e) $\epsilon = 2.0\text{m}^{-2}$, $\omega_s/\omega_1 = 4.0$; (f) $\epsilon = 2.0\text{m}^{-2}$, $\omega_s/\omega_1 = 5.0$.

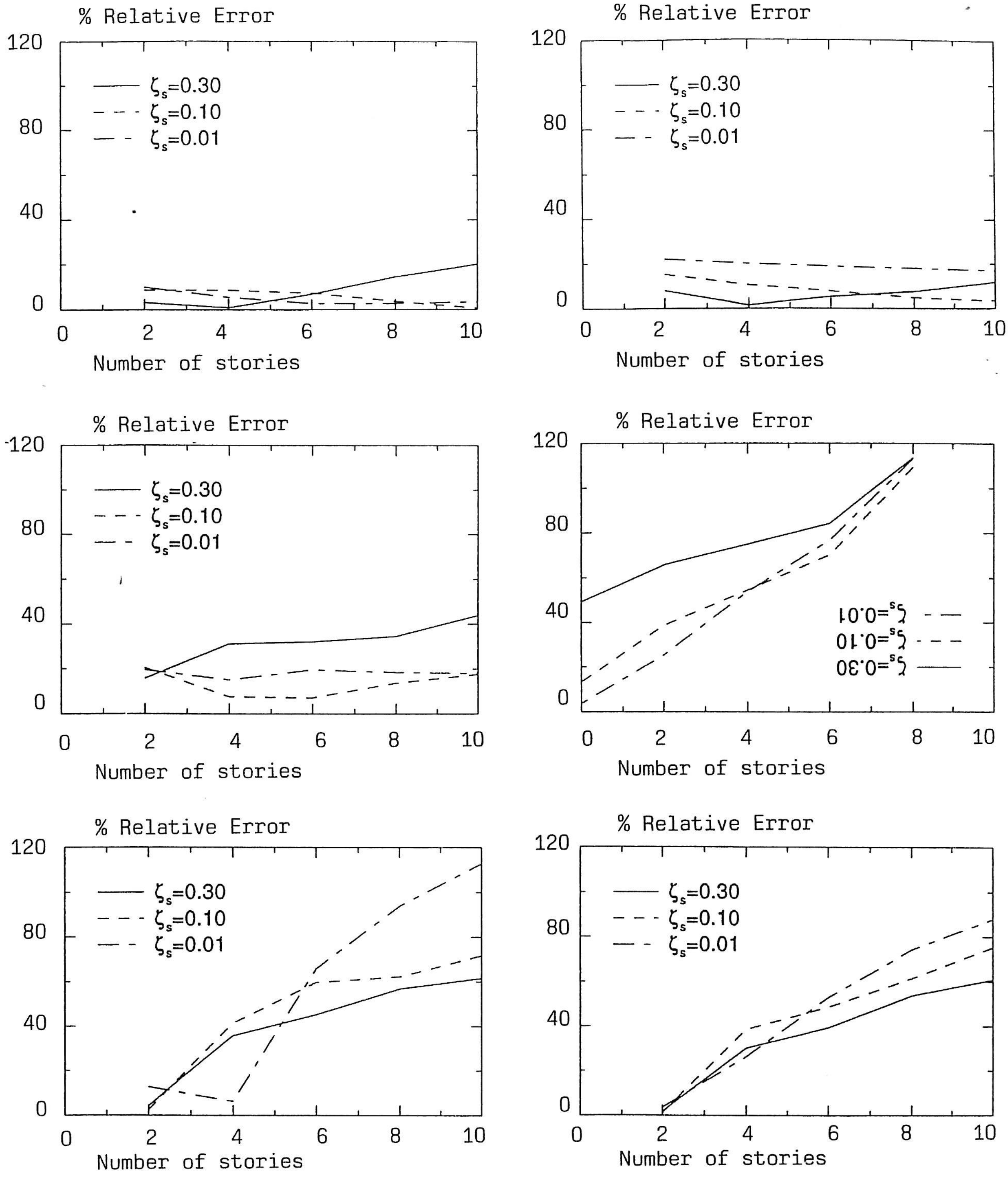

Figure 4: The percent relative error between exact and ESL solutions for the top-story displacement variance as a function of the number of shear-frame stories: severe nonlinearity (a) $\epsilon = 8.0\text{m}^{-2}$, $\omega_s/\omega_1 = 0.5$; (b) $\epsilon = 8.0\text{m}^{-2}$, $\omega_s/\omega_1 = 1.0$; (c) $\epsilon = 8.0\text{m}^{-2}$, $\omega_s/\omega_1 = 2.0$; (d) $\epsilon = 8.0\text{m}^{-2}$, $\omega_s/\omega_1 = 3.0$; (e) $\epsilon = 8.0\text{m}^{-2}$, $\omega_s/\omega_1 = 4.0$; (f) $\epsilon = 8.0\text{m}^{-2}$, $\omega_s/\omega_1 = 5.0$.

at the fundamental frequency of the equivalent linear structure, large responses are predicted, whereas the same input excitation does not have as dramatic an effect on the actual nonlinear system. Similar error results are observed for moderate nonlinearity and medium-band excitation, Figure 2(b). Whereas for moderate nonlinearity and narrowband excitation, Figure 2(c), errors are largest at $\omega_s/\omega_1 = 1$ and tend to decrease as ω_s/ω_1 increases.

From these results, it appears as though ESL provides approximations within 20% of the exact solutions for the following cases: $n = 4$, wide-band excitation; $n = 4$ and 6, medium-band excitation; $n = 4, 6, 8$ and 10, narrow-band excitation, $\omega_s/\omega_1 \geq 1$.

Considering now the case of severe nonlinearity, Figures 2(d)-2(f), it is immediately evident that the relative errors are of much larger magnitude. In addition, due to the increased stiffness with larger ϵ, the equivalent linear systems now have higher fundamental frequencies. As a result, the largest errors now occur at $\omega_s/\omega_1 = 3$. Moreover, in contrast to the case of moderate nonlinearity, where the errors tends to decrease as the bandwidth of the excitation becomes narrower, for the case of severe nonlinearity, the opposite trend is observed. In Figure 2(d), corresponding to wide-band excitation, it is observed that near the resonance excitation, $\omega_s/\omega_1 = 3$, the percent relative errors are approximately 7.5%$\times$ number of stories, i.e., for a 10−story structure the relative error is roughly 75%, for an 8−story structure roughly 60%, etc. For medium-band excitation, Figure 2(e), the largest errors increase to approximately 10%$\times$ number of stories. And for narrow-band excitation, Figure 2(f), the errors are further increased to nearly 12%$\times$ number of stories. Clearly, for all but a few values of ω_s/ω_1, equivalent statistical linearization of severely-nonlinear MDOF structures provides wholly inaccurate results.

Figures 3(a)-3(f) and Figures 4(a)-4(f) show the growth in the error as a function of the number of shear-frame stories for fixed values of ϵ and ω_s/ω_1. For moderate nonlinearity, Figures 3(a)-3(f), the error growth tends to be generally linear for wide-band and medium-band excitation when $\omega_s/\omega_1 \geq 2$, i.e., greater than or equal to the resonance condition, whereas the error is generally constant for narrow-band excitation. For values of ω_s/ω_1 less than two, the error growth is still linear for $\zeta_s = 0.30$ and generally constant for $\zeta_s = 0.10$ and $\zeta_s = 0.01$. Similar trends are noticed for the case of severe nonlinearity, Figures 4(a)-4(f), where now linear error growth occurs for $\omega_s/\omega_1 \geq 3$ for all bandwidths of excitation and relatively constant errors are observed for values of ω_s/ω_1 less than three.

5. CONCLUSIONS

Despite a few theoretical and practical shortcomings, equivalent statistical linearization remains a popular technique for analyzing MDOF nonlinear structures. The objective of this paper was to quantify the development of the error induced by using linearization for MDOF shear-frame structures. To this end, various combinations of structural parameters were considered in the analysis, yet considered in such a way that meaningful comparisons between structures of different dimensions were possible.

As is evident from the results presented in Figures 2-4, the relative error varies significantly with the degree of nonlinearity, the bandwidth of the excitation, and the ratio of the input excitation frequency to the first eigenfrequency of the corresponding linear structure. One recurring trend, however, is that the error grows approximately linearly with the number of degrees-of-freedom for values of $\frac{\omega_s}{\omega_1}$ greater than or equal to the resonance value. The slope of the increase varies from about $1\% - 12\%$ per story depending on the nonlinearity and excitation bandwidth. Moreover, while there are various parameter-specific cases for which equivalent linearization may give good statistical approximations for structures up to and surpassing ten degrees-of-freedom, in general, for cases of moderate nonlinearity, care should be used when applying equivalent linearization for structures with more than four degrees-of-freedom. In the case of severe nonlinearity, the general validity of equivalent linearization is further reduced to about two degrees-of-freedom, though there are cases where large errors are produced for even these low-dimensional structures.

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